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Teaching a substitution meaning for the equals sign in arithmetic contexts

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Background

Most children see the equals sign as meaning "write the answer here" rather than as a symbol of mathematical equivalence (Behr et al., 1976). This leads to inflexible thinking about arithmetic notation and causes difficulties when pupils meet symbolic algebra at the start of secondary school (Knuth et al., 2006). Recent studies have shown that teaching the equals sign means "is the same as" helps reduce pupils' difficulties (e.g. Li et al., 2008). Several interventions have demonstrated the value of presenting carefully selected sequences of non-canonical statements such as 7+7+9=14+9 and asking pupils to assess their truth (e.g. Molina et al., 2008). However, these studies neglect that symbolic substitution is central to understanding mathematical equivalence, and that the equals sign also means "can be exchanged for". We report a study that involved the development and evaluation of teaching materials that support both the exchanging and sameness meanings for the equals sign.

Research Questions

The project involved two phases. First, a design research phase to develop classroom materials that support both meanings for the equals sign in arithmetic contexts. This included the development of a specially designed piece of online mathematical software called "Sum Puzzles". There were also worksheets and flashcards to supplement the software. Second, a teaching intervention to Year 7 pupils to evaluate the classroom materials. In particular we were interested in how pupils came to engage with the exchanging meaning for the equals sign, and how this conflicted with their existing meanings.

Methods

The intervention involved two classes of Year 7 pupils for a period of four lessons each and was delivered by a member of the research team who is an experienced school teacher. Data was collected in the form of (i) log-files of pupils' performance with the Sum Puzzles software, (ii) field notes of the intervention lessons taken by a second member of the research team, and (iii) interviews with all pupils involved in the intervention to ascertain their understandings of the equals sign before and after the intervention.

Frame

The design of the software and classroom materials draws on Dörfler's (2006) "diagrammatic" approach to mathematical symbolising tasks. Most of the literature on pupils' conceptions of the equals sign adopt "structural" approaches (Kirshner, 2001) to notating task design in which meaning arises from the inter-relationships between symbols, as opposed to "referential" approaches, such as modeling, in which meaning is imported from external objects. In practice, however, structural approaches are often implicitly referential because the symbols stand for abstractions such as numbers and arithmetic principles (Dörfler, 2006). For example, to present 2+3=3+2 as a question of truthfulness is to ask for a comparison of the computational result referenced by each side. The designer intends such a statement to be a referent to the principle of commutativity, by which a learner might satisfy herself of its truth without calculating any results. Dörfler argued this implicit role of symbols as referents to otherwise inaccessible abstractions is a factor in many learners' sense of alienation. He suggested designers should consider tasks in which symbols and their transformations according to mathematical rules are themselves the focus of learning. Learning symbolic mathematics can then be seen as an exploratory, empirical activity in which learners make discoveries and test hypotheses. For example, if 2+3=3+2 is presented as a rule for making the substitutions 2+3→3+2 and 3+2→2+3 it becomes a reusable tool for transforming notation, rather than a closed question of truthfulness that is discarded once assessed. The commutative property of 2+3=3+2 is no longer a referenced abstraction, but can be observed as an exchange of numerals when the statement is used to transform arithmetic notation.

Research findings

The study demonstrated that pupils readily shift from focussing on computational results to working with the structure of arithmetic notation when engaged with the tasks. In particular their discussion during intervention lessons reflected a focus on the observable physical actions of using arithmetic equations to make exchanges of notation towards a specified task goal. Pupils tended to refer to the substitutive effects of a+b=b+a statements in terms of commutation ("swapping", "switching round" and so on) and, to a lesser extent, the substitutive effects of c=a+b statements in terms of partition ("splitting up", "separating" and so on). It was also found that when pupils were presented with prepared activities this focus on an exchanging meaning for the equals sign was exclusive: pupils did not concern themselves with the truthfulness of statements when using them to make exchanges of notation. However, when pupils were later challenged to make activities for their peers to work through they were forced to switch flexibly between both the sameness and exchanging meanings of the equals in order to succeed. This is quite challenging, and individual differences came to the fore in terms of success and performance at creating their own activities. There were also marked differences in the impact the intervention had on individual pupils' conceptions of the equals sign as meaning both "is the same as" and "can be exchanged for".

References

Behr, M., Erlwanger, S., & Nichols, E. (1976). How children view equality sentences (Report No. PMDC-TR-3). Tallahassee, Florida: Florida State University.

Dörfler, W. (2006). Inscriptions as objects of mathematical activities. In J. Maasz & W. Schloeglmann (Eds.), New Mathematics Education Research and Practice. Rotterdam/Taipei: Sense Publishers.

Kirshner, D. (2001). The structural algebra option revisited. In R. Sutherland, T. Rojano, A. Bell & R. Lins (Eds.), Perspectives on School Algebra (pp. 83-99). London: Kluwer Academic Publishers.

Knuth, E., Stephens, A., McNeil, N., & Alibali, M. (2006). Does understanding the equals sign matter? Evidence from solving equations. Journal for Research in Mathematics Education, 37, 297-312.

Li, X., Ding, M., Capraro, M., & Capraro, R. (2008). Sources of differences in children's understandings of mathematical equality: Comparative analysis of teacher guides and student texts in China and the United States. Cognition and Instruction, 26, 195-217.

Molina, M., Castro, E. & Castro, E. (2008), Third graders' strategies and use of relational thinking when solving number sentences. In S. Alatorre, T. Rojana, O. Figueras, J. Cortina & A. Sepulveda (Eds.), Proceedings of the 32nd Conference of the International Group for the Psychology of Mathematics Education (Vol. 3, pp. 399-406). Morelia, Mexico: IGPME.